

# Naked Black Holes

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## Abstract

It is shown that there are large static black holes for which all curvature invariants are small near the event horizon, yet any object which falls in experiences enormous tidal forces *outside* the horizon. These black holes are charged and near extremality, and exist in a wide class of theories including string theory. The implications for cosmic censorship and the black hole information puzzle are discussed.

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## 1. Introduction

It is commonly believed that the spacetime curvature is small near the horizon of a large static black hole, and objects can fall in without being disrupted. We will show that this is not always the case. There are black holes in which the area of the event horizon is large and all curvature invariants are small near the horizon. Nevertheless, any object which falls in experiences enormous tidal forces outside the horizon. This is a result of the fact that the curvature is actually very large and almost null near the horizon. When measured in a static frame (which is also becoming null), the components of the curvature remain small. This implies that all curvature invariants are small, and perturbative  $\alpha'$  corrections in string theory are negligible. However, in a freely falling frame, the curvature components are very large. Since the region of large tidal forces is visible to distant observers, we will call such objects “naked black holes”.

These black holes exist in a wide class of theories including the supergravity theories that arise in the low energy limit of string theory. Our examples are all charged black holes which are either at or near extremality. In fact, many of the solutions to string theory which have been found in recent years [1] (including the higher dimensional black  $p$ -branes) have limits in which they become naked in the above sense. Some of these solutions are known to have the property that the horizon shrinks down to zero size and becomes singular as one approaches the extremal limit (for fixed mass). We will show that the curvature felt by an infalling observer can become large even when the area of the horizon remains large.

In retrospect, it is not surprising that there can be a significant difference between the size of the curvature seen by static and freely falling observers. After all, static observers measure the curvature in the rest frame of the black hole. Near the horizon, freely falling observers are highly boosted with respect to the static ones, and one would expect their curvature components to be much larger. From this viewpoint, it is surprising that the familiar Schwarzschild and Reissner-Nordström black holes have approximately the same size curvature in the static and freely falling frames. This is possible only because of a special cancellation between different components of the curvature in these metrics: Their curvature is actually invariant under radial boosts. Based only on these two examples, one might have concluded that boost invariance of the curvature was somehow implied by the structure of the event horizon. We will see that this is not the case.

The existence of naked black holes has implications for cosmic censorship [2]. Although

they do not affect a strict interpretation of this conjecture in terms of singularities, they weaken the spirit of cosmic censorship. One of the motivations for this conjecture was to show that general relativity would break down only in regions of spacetime shielded by event horizons. While naked black holes have nonsingular event horizons, the curvature outside can be larger than the Planck scale. Thus, effects of quantum gravity could be visible outside macroscopic black holes. Although we will consider the eternal static black holes, it seems likely that one can form the near extremal black holes from regular initial data.

Naked black holes may also play a role in resolving the black hole information puzzle. Recently, the entropy of certain extremal and near extremal black holes has been reproduced in string theory by counting states in the limit of weak coupling [3,4,5]. Furthermore, the radiation produced at weak coupling has the same spectrum as the Hawking radiation emitted by the black hole [6,7]. However, the weak coupling description is manifestly unitary, while Hawking has given arguments that black hole radiation is not unitary [8,9]. Of course a basic assumption in his arguments is that matter can fall into a large black hole undisturbed, which makes it difficult to see how the information about the state of the matter gets out. We will see that some of the near extremal black holes whose entropy has been understood by counting string states are, in fact, naked. Matter falling into such black holes will be disrupted by the large tidal forces, which may invalidate the simple argument for information loss in these cases.

In the next section, we discuss the difference between the curvature in a static frame and in an infalling frame for a general class of metrics. Section 3 contains some examples of naked black holes in general relativity and section 4 contains examples from string theory. Some implications of the existence of these black holes are discussed in section 5.

## 2. Curvature and Tidal Forces

We begin by considering the following class of metrics in  $d$  spacetime dimensions,

$$ds^2 = -\frac{F(r)}{G(r)}dt^2 + \frac{dr^2}{F(r)} + R^2(r)d\Omega_{n+1} + H^2(r)dy^i dy_i, \quad (2.1)$$

where  $i = n + 3, \dots, d - 1$ . This class includes most of the recently discussed black hole and black  $p$ -brane ( $p = d - n - 3$ ) solutions; the metric will have a horizon at  $r = r_0$  if  $F(r_0) = 0$ . The curvature is, of course, completely characterized by the components of the

Riemann tensor in an orthonormal frame. Let us consider first the static frame

$$\begin{aligned}
(e_0)_\mu &= -F^{1/2}(r)G^{-1/2}(r) \partial_\mu t, & (e_1)_\mu &= F^{-1/2}(r) \partial_\mu r, \\
(e_a)_\mu &= R(r) \sin \theta_1 \dots \sin \theta_{a-2} \partial_\mu \theta_{a-1}, \\
(e_{n+2})_\mu &= R(r) \sin \theta_1 \dots \sin \theta_n \partial_\mu \phi, \\
(e_i)_\mu &= H(r) \partial_\mu y_i,
\end{aligned} \tag{2.2}$$

where  $\theta_{a-1}$ ,  $a = 2, \dots, n+1$  and  $\phi$  are coordinates on  $S^{n+1}$ . The only non-vanishing components of the curvature in this orthonormal frame are  $R_{0101}$ ,  $R_{0k0k}$  (no sum on  $k$ ),  $R_{1k1k}$ , and  $R_{klkl}$ , where  $k, l = 2, \dots, d-1$  (and components related to these by symmetry). As we will see below, radially infalling observers will measure the curvature not in this static frame, but in terms of another orthonormal frame related to (2.2) by a local radial boost. That is, a frame where

$$(e_{0'})_\mu = \cosh \alpha (e_0)_\mu + \sinh \alpha (e_1)_\mu, \quad (e_{1'})_\mu = \sinh \alpha (e_0)_\mu + \cosh \alpha (e_1)_\mu, \tag{2.3}$$

the other  $(e_k)_\mu$  are as before, and  $\alpha = \alpha(x^\mu)$  is some function of the coordinates.

The components of the curvature in any boosted frame cannot be smaller than the components in the static frame. This can be seen as follows. The non-vanishing components of the curvature in the boosted frame will be

$$\begin{aligned}
R_{0'1'0'1'} &= R_{0101}, & R_{0'k1'k} &= \cosh \alpha \sinh \alpha (R_{0k0k} + R_{1k1k}), \\
R_{0'k0'k} &= R_{0k0k} + \sinh^2 \alpha (R_{0k0k} + R_{1k1k}), \\
R_{1'k1'k} &= R_{1k1k} + \sinh^2 \alpha (R_{0k0k} + R_{1k1k}),
\end{aligned} \tag{2.4}$$

and  $R_{klkl}$ . For each  $k$ , consider the larger of the two components  $R_{0k0k}$ ,  $R_{1k1k}$ . Since  $R_{0k0k} + R_{1k1k}$  has the same sign as the larger component, it is clear that its magnitude cannot decrease under a boost. The curvature can remain unchanged if  $R_{0k0k} = -R_{1k1k}$ ; this occurs, for example, in the Schwarzschild solution. In general, the boosted components are larger and the curvature is minimized in the static frame. (The static frame is preferred because it is the only one for which the  $R_{0k1k}$  components vanish.) The static frame is thus the most convenient one for calculating curvature invariants and contractions of the Riemann tensor. (In the examples we consider, the size of  $R_{0k0k}$ ,  $R_{1k1k}$  in the static frame is comparable to that of  $R_{0101}$ ,  $R_{klkl}$ .)

To determine the physical effect of the curvature on geodesic observers, we need to use an orthonormal frame which is parallelly propagated along the geodesics. If we consider

timelike geodesics in the metric (2.1), with proper time  $\tau$  and tangent vector  $u^\mu = dx^\mu/d\tau$ , we can choose coordinates on  $S^{n+1}$  so that the geodesic lies in the equatorial plane. There are a number of constants of motion:

$$E = \frac{F(r)}{G(r)}\dot{t}, \quad p_i = H^2(r)\dot{y}_i, \quad p_\phi = R^2(r)\dot{\phi}, \quad (2.5)$$

where a dot denotes  $d/d\tau$ . For the sake of simplicity, we will consider radial geodesics,  $p_i = p_\phi = 0$ . From the normalization condition  $u^\mu u_\mu = -1$ , we can see that

$$\dot{r}^2 = E^2 G(r) - F(r). \quad (2.6)$$

The parallelly propagated orthonormal frame, in which  $(e_{0'})_\mu = u_\mu$ , is then related to the static frame by a radial boost,

$$\begin{aligned} (e_{0'})_\mu &= u_\mu = -E\partial_\mu t + \frac{\dot{r}}{F(r)}\partial_\mu r \\ &= \cosh \alpha (e_0)_\mu + \sinh \alpha (e_1)_\mu, \end{aligned} \quad (2.7)$$

and

$$(e_{1'})_\mu = \sinh \alpha (e_0)_\mu + \cosh \alpha (e_1)_\mu, \quad (2.8)$$

where  $\cosh \alpha = E[G(r)/F(r)]^{1/2}$ . Note that since the horizon lies at  $F(r) = 0$ , the boost parameter  $\alpha$  diverges as we approach the horizon.

We can compute the components of the curvature in this frame by first computing the components in the static frame, and then applying the transformations (2.4). However, there is another, simpler route to computing the boosted components, which offers a more direct physical understanding. We can see from (2.4) that the difference between the static frame and the boosted frame is essentially the same for all components, so it suffices to calculate  $R_{0'k0'k}$ . These components correspond to tidal forces in the transverse directions. In other words, they measure the relative acceleration of nearby geodesics. In fact, they are simply given by

$$R_{0'a0'a} = -\frac{\ddot{R}}{R}, \quad R_{0'i0'i} = -\frac{\ddot{H}}{H}, \quad (2.9)$$

as we now show. For a family of radial infalling geodesics with tangent vector  $u^\mu$ , and the set of deviation vectors  $\eta = \partial/\partial\theta_{a-1}$  for  $a = 2, \dots, n+1$ , and  $\eta = \partial/\partial\phi$  for  $a = n+2$ , we have

$$u^\nu \nabla_\nu \eta^\sigma = u^\nu \Gamma_{\nu\rho}^\sigma \eta^\rho = u^r \frac{R'}{R} \eta^\sigma = \frac{\dot{R}}{R} \eta^\sigma. \quad (2.10)$$

The geodesic deviation equation then implies

$$R_{\mu\nu\rho}{}^{\sigma} u^{\mu} \eta^{\nu} u^{\rho} = -u^{\mu} \nabla_{\mu} (u^{\nu} \nabla_{\nu} \eta^{\sigma}) = -\frac{\ddot{R}}{R} \eta^{\sigma}. \quad (2.11)$$

Thus

$$\begin{aligned} R_{0'a0'a} &= R_{\mu\nu\rho}{}^{\sigma} u^{\mu} (e_a)^{\nu} u^{\rho} (e_a)_{\sigma} = -\frac{\ddot{R}}{R} = -\frac{1}{R} (R'' \dot{r}^2 + R' \ddot{r}) \\ &= -\frac{1}{R} \left[ R'' (E^2 G - F) + \frac{R'}{2} (E^2 G' - F') \right], \end{aligned} \quad (2.12)$$

where we have used (2.6). Similarly,  $R_{0'i0'i} = -\ddot{H}/H$  for  $i = n+3, \dots, d-1$ . The terms proportional to  $E^2$  in this expression correspond to the enhancement of the curvature in the geodesic frame over the static frame. Note that although the boost parameter diverges at the horizon, these terms will generally be finite. This is due to a cancellation between the leading order contributions to  $R_{0k0k}$  and  $R_{1k1k}$  near the horizon. For Schwarzschild, there is complete cancellation; the terms proportional to  $E^2$  vanish because  $R'' = G' = 0$ . It is clear that whenever these terms do not vanish, the tidal force can be made arbitrarily large, simply by taking the conserved energy per unit mass along the geodesic  $E$  to be large. But this is also true for nonradial geodesics in Schwarzschild. Conversely, no matter how large the tidal force is, we can find a family of geodesics for which its effect is small simply by decreasing  $E$ . To avoid this ambiguity, we will always assume the energy per unit mass  $E$  is of order one, i.e. we will consider geodesics that start at infinity with small velocity. When we calculate  $R_{0'k0'k}$  in the examples, we will only keep the part proportional to  $E^2$ , which represents the difference between the static frame and the boosted frame.

### 3. Examples from General Relativity

In this section we discuss two examples of black holes with large horizon area and small curvature components in the static frame, but large curvature in a freely falling frame. Both examples are four dimensional black holes in general relativity coupled to gauge fields and scalar fields. The solutions will take the familiar form

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + R^2(r)d\Omega_2. \quad (3.1)$$

For metrics of this type, the difference between the curvature in the static and infalling frames can be easily seen as follows. The event horizon is at  $r = r_+$ , the largest value of

$r$  for which  $F$  vanishes, and it has area  $4\pi R(r_+)^2$ . The static curvature near the horizon contains the components

$$R_{2323} = \frac{1}{R^2}, \quad R_{0202} = \frac{F'R'}{2R}. \quad (3.2)$$

However, the requirement of positive energy density  $G_{00} \geq 0$ , implies that, near the horizon,  $1/R^2 \geq F'R'/R$ . So the static curvature will be small (in Planck units) if  $R(r_+) \gg 1$ . The curvature in a freely falling frame is given by (2.12) with  $G(r) = 1$ ,

$$R_{0'20'2} = -\frac{1}{R} \left[ R''(E^2 - F) - \frac{F'R'}{2} \right]. \quad (3.3)$$

Near the horizon,  $F(r)$  is small, so this will be larger than the Planck scale if  $|R''/R| > 1$ . Thus to satisfy our conditions, we simply require  $R \gg 1$  and  $|R''/R| > 1$  at  $r = r_+$ . In the examples below, the tidal forces are of the same magnitude over a radial proper distance of order  $R(r_+)$  outside the horizon. Thus,  $R(r_+)$  is a measure of the size of the region of large tidal forces.

### 3.1. Dilaton Black Holes

Our first example is the dilaton black hole metrics [10,11]. These are solutions of a theory with a single Maxwell field  $F_{\mu\nu}$  and scalar field  $\phi$  with the coupling between the Maxwell field and the scalar field governed by an arbitrary constant  $a$ . The action is

$$S = \int d^4x \sqrt{-g} [R - 2(\nabla\phi)^2 - e^{-2a\phi} F_{\mu\nu} F^{\mu\nu}], \quad (3.4)$$

and the metric for a dilaton black hole is given by (3.1) with

$$F(r) = \frac{(r - r_+)(r - r_-)}{R^2} \quad (3.5)$$

and

$$R(r) = r \left( 1 - \frac{r_-}{r} \right)^{a^2/(1+a^2)}. \quad (3.6)$$

There is a horizon at  $r = r_+$  and a singularity at  $r = r_-$  for  $a \neq 0$ . For  $a = 0$ , this metric reduces to the Reissner-Nordström metric;  $r = r_-$  is an inner horizon, and there is a singularity at  $r = 0$ . The extremal limit in both cases is  $r_+ = r_-$ . The ADM mass and charge are

$$\begin{aligned} M &= \frac{r_+}{2} + \left( \frac{1 - a^2}{1 + a^2} \right) \frac{r_-}{2}, \\ Q &= \left( \frac{r_+ r_-}{1 + a^2} \right)^{1/2}. \end{aligned} \quad (3.7)$$

The horizon area will be large and the static curvature will be small (in Planck units) if

$$R(r_+) = r_+ \epsilon^{a^2/(1+a^2)} \gg 1, \quad (3.8)$$

where  $\epsilon \equiv (1 - r_-/r_+)$ . Note that the exponent of  $\epsilon$  is always less than one. As discussed above, the tidal forces in the geodesic frame will be larger than the Planck scale if

$$\left| \frac{R''}{R} \right| = \frac{a^2}{(1+a^2)^2} \frac{(1-\epsilon)^2}{r_+^2 \epsilon^2} > 1. \quad (3.9)$$

This will be satisfied, for  $a \neq 0$ , if  $r_+ \epsilon \ll 1$ . Thus we see that there is a range of parameters for which the curvature in the static frame is small, but infalling observers experience large tidal forces near the horizon, namely  $\epsilon \ll 1$  and  $\epsilon^{-a^2/(1+a^2)} \ll r_+ \ll \epsilon^{-1}$ . Physically, the reason for the difference in the size of the curvature components is that the curvature is becoming large and nearly null near the horizon. Since the static frame is becoming null, it does not see this effect. Since  $\epsilon$  is small, these black holes are all close to extremality, and since  $r_+$  is large, they have a large mass. For fixed mass, the area of the event horizon goes to zero in the extremal limit. The spacetime develops a null singularity if  $0 < a \leq 1$  and a timelike singularity if  $a > 1$ . We are considering a different limit in which the mass is increased as one approaches extremality, so the horizon area remains large.

One might wonder whether infalling observers will experience large tidal forces only in cases where the extremal limit is singular. This seems to be suggested by the above example, since the one case where the extremal limit is nonsingular, the Reissner-Nordström metric ( $a = 0$ ), is also the one case where the tidal forces remain small for infalling observers. However, this is not the case, as the next example shows.

### 3.2. $U(1)^2$ black holes

Our second example is a class of black hole solutions to a theory with two Maxwell fields and a scalar field [10,12]. The action is

$$S = \int d^4x \sqrt{-g} [R - 2(\nabla\phi)^2 - e^{-2\phi}(F_{\mu\nu}F^{\mu\nu} + G_{\mu\nu}G^{\mu\nu})], \quad (3.10)$$

and the metric for the black hole solutions is again given by (3.1) with

$$F(r) = \frac{(r - r_+)(r - r_-)}{R^2}, \quad (3.11)$$

but now

$$R^2(r) = r^2 - \Sigma^2. \quad (3.12)$$



This metric has an event horizon at  $r = r_+$ , an inner horizon at  $r = r_-$ , and a singularity at  $r = |\Sigma|$ . One Maxwell field has an electric charge  $Q$  and the other has a magnetic charge  $P$ . In terms of the mass and charges,

$$\begin{aligned}\Sigma &= \frac{P^2 - Q^2}{2M}, \\ r_{\pm} &= M \pm (M^2 + \Sigma^2 - Q^2 - P^2)^{1/2}.\end{aligned}\tag{3.13}$$

If  $Q = P$ , then  $\Sigma = 0$  and this metric is just the Reissner-Nordström metric. If one of the charges vanishes, then  $|\Sigma| = r_-$  and it reduces to the dilaton black hole metric discussed in the previous section with  $a = 1$ . With both charges nonzero,  $|\Sigma| < r_-$ , and there is a smooth horizon in the extremal limit  $r_+ = r_-$ .

As before, the horizon area will be large and the static curvature small if

$$R(r_+) = r_+ \delta^{1/2} \gg 1,\tag{3.14}$$

where  $\delta \equiv (1 - \Sigma^2/r_+^2)$ . The curvature in the geodesic frame will be larger than the Planck scale if

$$\left| \frac{R''}{R} \right| = \frac{(1 - \delta)}{r_+^2 \delta^2} > 1,\tag{3.15}$$

at  $r = r_+$ . There is again a range of parameters for which both these conditions are satisfied, namely  $\delta \ll r_+^2 \delta^2 \ll 1$ . Thus, even though the area of the event horizon is large even at extremality, observers experience large tidal forces outside the black hole.

Since  $|\Sigma| < r_-$ ,  $(1 - r_-/r_+) < \delta$ , and thus the above condition implies that the black hole must be near-extreme for this effect to appear. Further, since both  $\Sigma$  and  $r_-$  are close to  $r_+$ , they must be close to each other. This implies that one charge is much greater than the other, so away from the horizon, this solution resembles the  $a = 1$  dilaton black hole discussed above.

#### 4. Examples from String Theory

In this section, we will consider black hole and black  $p$ -brane solutions which arise in string theory. We will consider both the Einstein and the string metrics and show that in both cases, the tidal forces experienced by infalling objects can be large when the horizon area is large. The metrics we discuss are solutions of the low energy equations of motion, to leading order in  $\alpha'$ . Since we are interested in situations where the components of the

curvature in a freely falling frame become large, one might worry that  $\alpha'$  corrections will become important. This is not the case. The equations of motion take the general form

$$R_{\mu\nu} + \text{matter contributions} + \alpha' R_{\mu}{}^{\rho\sigma\lambda} R_{\nu\rho\sigma\lambda} + \dots = 0. \quad (4.1)$$

We can compare the size of the  $\alpha'$  correction to the leading order term in any frame we choose, since under a boost, both quantities will be equally boosted. In particular, if the curvature in the static frame is small, so that the curvature squared term is small compared to the first term, then in the geodesic frame, there will be cancellations in the calculation of the  $\alpha'$  term which make it the same relative size as in the static frame. We will see that one can also choose the parameters so that  $ge^{\phi}$  is small in the region where the tidal forces become large, so perturbative quantum corrections should be small as well.

In the remainder of this section, we will set  $\alpha' = 1$ , so we are working in string units.

#### 4.1. Neveu-Schwarz charged black holes

We first consider the black hole solution with electric Neveu-Schwarz charges associated with internal momentum and string winding number. The string metric is [13]

$$ds^2 = -\Delta^{-1} \left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega_2, \quad (4.2)$$

where

$$\Delta = \left(1 + \frac{r_0 \sinh^2 \gamma_1}{r}\right) \left(1 + \frac{r_0 \sinh^2 \gamma_p}{r}\right) \quad (4.3)$$

and the dilaton is given by

$$e^{2\phi} = \Delta^{-1/2}. \quad (4.4)$$

The ADM mass of these black holes is

$$M = \frac{r_0 R V}{g^2} (2 + \cosh 2\gamma_1 + \cosh 2\gamma_p), \quad (4.5)$$

and the integer normalized charges are

$$n = \frac{R^2 V}{g^2} r_0 \sinh 2\gamma_p, \quad m = \frac{V}{g^2} r_0 \sinh 2\gamma_1, \quad (4.6)$$

where  $R$  is the radius of a compact internal direction, and  $(2\pi)^5 V$  is the volume of an internal five-torus. (We are using the same conventions as [5].)

The curvature in the static frame is of order  $1/r_0^2$  at the horizon  $r = r_0$ , so we must take  $r_0 \gg 1$  to keep it small. The curvature in the infalling frame can be found from (2.12) with  $F(r) = (1 - r_0/r)$ ,  $G(r) = \Delta$ , and  $R(r) = r$ . At  $r = r_0$ , it is

$$R_{0'20'2} = -\frac{R'}{2R} (G'E^2 - F') = -\frac{E^2 \Delta'}{2r_0} + \frac{1}{2r_0^2}. \quad (4.7)$$

Since

$$\Delta'(r_0) = -\frac{1}{r_0} (\sinh^2 \gamma_1 \cosh^2 \gamma_p + \cosh^2 \gamma_1 \sinh^2 \gamma_p), \quad (4.8)$$

we can make the tidal forces arbitrarily large by increasing  $\gamma_1$  or  $\gamma_p$ . Physically, this just corresponds to increasing the mass and charges.

In the regime where the tidal forces are large,  $e^\phi$  is very small, and thus perturbative quantum corrections are negligible. String  $\alpha'$  corrections associated with powers of the curvature are also negligible since the static curvature is small. Since the dilaton  $\phi$  is large and negative, one might worry about  $\alpha'$  corrections involving derivatives of the dilaton. However, near  $r = r_0$ ,

$$\partial_r \phi = -\frac{1}{4} \frac{\Delta'}{\Delta} \sim \frac{1}{r_0} \quad (4.9)$$

when  $\gamma_1$  or  $\gamma_p$  is large. Thus,  $\alpha'$  corrections involving the dilaton are also unimportant.

The extremal limit for this class of black holes is  $r_0 \rightarrow 0$ ,  $\gamma_1, \gamma_p \rightarrow \infty$  with  $n, m$  fixed. It may appear that the large tidal forces are present far from the extremal limit, since we have taken  $r_0 \gg 1$ . However, for fixed charges, the mass above extremality is

$$\Delta M = M - M_{ext} \approx \frac{2r_0 R V}{g^2}. \quad (4.10)$$

When  $r_0 \gg 1$ ,  $\Delta M$  is large, but  $\Delta M/M$  is still small since  $\gamma_1$  or  $\gamma_p$  is large.

The Einstein metric is obtained by multiplying (4.2) by  $e^{-2\phi}$ . The area of the horizon is thus increased by the factor  $\cosh \gamma_1 \cosh \gamma_p$  and the static frame curvature near the horizon is decreased by the same factor. The Einstein metric takes the form (3.1) and one can verify that the tidal forces in this metric are proportional to  $1/r_0$ . Thus, for the range of parameters we have been considering, the tidal forces would not be large for infalling observers. However, if one takes  $r_0$  small and  $\gamma_1, \gamma_p$  sufficiently large, then both the size of the black hole and the tidal forces in the Einstein metric will be large. This example is closely related to the examples of the previous section. If  $\gamma_1 = \gamma_p$ , the Einstein metric is the same as the black hole discussed in section 3.1 with  $a = 1$  (or the one discussed in 3.2 with  $P = 0$ ). If  $\gamma_1 = 0$  or  $\gamma_p = 0$ , the metric is the same as the one in 3.1 with  $a = \sqrt{3}$ .

#### 4.2. $p$ -branes with a Ramond-Ramond charge

The effect we have been discussing is also present for extended objects in string theory. We now consider black  $p$ -branes with a single Ramond-Ramond charge in  $d = 10$  [14]. The string metric is

$$ds^2 = f^{-1/2} \left[ - \left( 1 - \frac{r_0^n}{r^n} \right) dt^2 + dy^i dy_i \right] + f^{1/2} \left[ \left( 1 - \frac{r_0^n}{r^n} \right)^{-1} dr^2 + r^2 d\Omega_{n+1} \right], \quad (4.11)$$

where there are  $p = 7 - n$  coordinates  $y^i$ , and

$$f = 1 + \frac{r_0^n \sinh^2 \alpha}{r^n}. \quad (4.12)$$

The dilaton is given by  $e^{2\phi} = f^{(n-4)/2}$ . The charge is

$$Q \sim \frac{r_0^n}{g} \sinh 2\alpha. \quad (4.13)$$

If we assume that the longitudinal coordinates  $y_i$  are periodically identified, so the  $p$ -brane is wrapped around a torus with volume  $V$ , the mass is

$$M \sim \frac{r_0^n V}{g^2} \left( \frac{n+2}{n} + \cosh 2\alpha \right), \quad (4.14)$$

The curvature in the static frame is of order  $1/(r_0^2 \cosh \alpha)$  at the horizon  $r = r_0$ . The curvature in the geodesic frame can again be found from (2.12) with  $F(r) = f^{-1/2}(1 - r_0^n/r^n)$ ,  $G(r) = 1$ ,  $R(r) = f^{1/4}r$  and  $H(r) = f^{-1/4}$

$$\begin{aligned} R_{0'a0'a} &= -\frac{1}{R} \left[ R'' (E^2 - F) - \frac{R' F'}{2} \right] = \frac{E^2}{4f^2} \left( \frac{3}{4} f'^2 - f f'' - \frac{2}{r} f f' \right) - \dots \\ &= \frac{E^2}{16\tilde{f}^2 r^2} (4n - n^2) + \dots \end{aligned} \quad (4.15)$$

for  $a = 2, \dots, n+2$ , and

$$R_{0'i0'i} = -\frac{1}{H} \left[ H'' (E^2 - F) - \frac{H' F'}{2} \right] = \frac{E^2}{4f^2} \left( -\frac{5}{4} f'^2 + f f'' \right) + \dots = \frac{E^2}{16\tilde{f}^2 r^2} (4n - n^2) + \dots \quad (4.16)$$

for  $i = n+3, \dots, 9$ , where

$$\tilde{f} = 1 + \frac{r^n}{r_0^n \sinh^2 \alpha}. \quad (4.17)$$

and we have kept only the terms proportional to  $E^2$  on the right hand side. Notice that the leading order behaviors of  $R_{0'a0'a}$  and  $R_{0'i0'i}$  are exactly the same.

Let us first consider the extremal solutions, which describe D-branes [15] at strong coupling. The extremal limit is  $r_0 \rightarrow 0$ ,  $\alpha \rightarrow \infty$  with  $Q$  fixed. For  $n < 4$ , the curvature of the extremal solution in the static frame diverges as  $r \rightarrow 0$ , which is similar to the dilaton black hole in which the horizon has become singular in the extremal limit. For  $n = 4$  (i.e., the threebrane), the curvature and the area of the 5-spheres approach constant values as  $r \rightarrow 0$ . Finally, for  $n > 4$ , the static frame curvature of the extremal solution *vanishes* as  $r \rightarrow 0$ . This is related to the fact that at small  $r$ ,  $R(r) \sim r^{1-n/4}$ , so the area of the  $n+1$ -spheres diverges as  $r \rightarrow 0$ . Since timelike geodesics reach  $r = 0$  in finite proper time, this raises the question of whether the spacetime can be extended beyond this boundary. A smooth extension is known [16] for the threebrane ( $n = 4$ ), but not for any of the lower branes.

It is clear from (4.15), (4.16) that no such extension is possible. For all  $n \neq 4$ , the tidal forces diverge like  $1/r^2$  as  $r \rightarrow 0$ . Thus, the solutions with  $n > 4$ , which seem non-singular from the point of view of the static frame curvature, are actually singular. (This could also have been inferred from the fact that the area of the spheres diverges in a finite proper time along a timelike geodesic, since this implies that the separation between two such geodesics diverges in finite proper time, and hence the tidal forces must diverge along the geodesic.) Since the static frame curvature of these solutions vanishes as  $r \rightarrow 0$ , the  $\alpha'$  corrections vanish, and so these solutions should still be singular when all  $\alpha'$  corrections are included. This is similar to the singular gravitational plane wave solutions discussed in [17]. For  $n = 4$ , the tidal forces remain finite at  $r = 0$ , as required by the existence of a smooth extension.

Now we consider the non-extreme solutions. For any  $n$ , the curvature near the horizon  $r = r_0$  in the static frame is small if  $r_0^2 \cosh \alpha \gg 1$ . This insures that the area of the  $r = r_0$  sphere at constant  $y_i$  is large. The total area of the horizon is proportional to the volume  $V$ , so it can be made large by a suitable choice of  $V$ . On the other hand, the tidal forces will be large, for  $n \neq 4$ , if  $r_0 \ll 1$ . We can satisfy this in conjunction with the other conditions simply by taking  $\alpha$  sufficiently large. As before, this corresponds to a near extremal configuration.

#### 4.3. Solutions with several charges

As our final example, we consider the ten dimensional solution with three charges

which gives, on dimensional reduction, the five-dimensional black holes whose entropy [3,4,5] and Hawking radiation [6,7] has been explained in terms of an effective string picture derived from D-brane calculations.

The string metric for this solution is [18,19]

$$ds^2 = f_1^{-1/2} f_5^{-1/2} \left[ -dt^2 + dx_5^2 + \frac{r_0^2}{r^2} (\cosh \sigma dt + \sinh \sigma dx_5)^2 \right] + f_1^{1/2} f_5^{-1/2} dx_i dx^i \\ + f_1^{1/2} f_5^{1/2} \left[ \left( 1 - \frac{r_0^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3 \right], \quad (4.18)$$

where  $i = 6, 7, 8, 9$ ,

$$f_1(r) = 1 + \frac{r_1^2}{r^2}, \quad f_5(r) = 1 + \frac{r_5^2}{r^2}, \quad (4.19)$$

and  $r_1 = r_0 \sinh \alpha$ ,  $r_5 = r_0 \sinh \gamma$ . The dilaton is  $e^{2\phi} = f_1/f_5$ , and the integer charges are

$$Q_1 = \frac{V r_0^2}{2g} \sinh 2\alpha, \quad (4.20)$$

$$Q_5 = \frac{r_0^2}{2g} \sinh 2\gamma, \quad (4.21)$$

and

$$n = \frac{R^2 V r_0^2}{2g^2} \sinh 2\sigma, \quad (4.22)$$

where the volume in the 6789 directions is  $(2\pi)^4 V$ , and the radius of the 5 direction is  $R$ .

The mass is

$$M = \frac{R V r_0^2}{2g^2} (\cosh 2\alpha + \cosh 2\gamma + \cosh 2\sigma). \quad (4.23)$$

We will also set  $r_n = r_0 \sinh \sigma$ . It is also convenient to define

$$F(r) = f_1^{-1/2} f_5^{-1/2} \left( 1 - \frac{r_0^2}{r^2} \right), \quad (4.24)$$

$$G(r) = f_n = 1 + \frac{r_n^2}{r^2}, \quad (4.25)$$

$$R(r) = f_1^{1/4} f_5^{1/4} r \text{ and } H(r) = f_1^{1/4} f_5^{-1/4}. \quad (4.26)$$

This metric is not of the form (2.1), because of the off-diagonal terms between  $t$  and  $x_5$ ; however, it is relatively straightforward to generalize the analysis given for (2.1) to this

case. We pick an orthonormal frame

$$\begin{aligned}
(e_0)_\mu &= -f_1^{-1/4} f_5^{-1/4} f_n^{-1/2} \left(1 - \frac{r_0^2}{r^2}\right)^{1/2} \partial_\mu t, \\
(e_1)_\mu &= f_1^{1/4} f_5^{1/4} \left(1 - \frac{r_0^2}{r^2}\right)^{-1/2} \partial_\mu r, \\
(e_2)_\mu &= f_1^{1/4} f_5^{1/4} r \partial_\mu \theta_1, \quad (e_3)_\mu = f_1^{1/4} f_5^{1/4} r \sin \theta_1 \partial_\mu \theta_2, \\
(e_4)_\mu &= f_1^{1/4} f_5^{1/4} r \sin \theta_1 \sin \theta_2 \partial_\mu \phi, \\
(e_5)_\mu &= f_1^{-1/4} f_5^{-1/4} \left[ f_n^{-1/2} \frac{r_0^2}{r^2} \cosh \sigma \sinh \sigma \partial_\mu t + f_n^{1/2} \partial_\mu x_5 \right], \\
(e_i)_\mu &= f_1^{1/4} f_5^{-1/4} \partial_\mu x_i,
\end{aligned} \tag{4.27}$$

where  $i = 6, \dots, 9$ . The non-vanishing components of the curvature in this orthonormal frame are  $R_{0101}$ ,  $R_{0k0k}$ ,  $R_{0151}$ ,  $R_{0k5k}$ ,  $R_{1k1k}$ , and  $R_{klkl}$ , where  $k = 2, \dots, 9$ . Since  $R_{0k1k} = 0$ , this frame will still minimize the curvature components under radial boosts.

The timelike geodesics in this metric have constants of motion  $E$ ,  $p_5$ ,  $p_i$ , and  $p_\phi$ . We can choose coordinates on the three-sphere so that the geodesic lies in the equatorial plane, so the tangent vector to the geodesics is

$$u_\mu = \left( -E, \frac{\dot{r}}{F(r)}, 0, 0, p_\phi, p_5, p_i \right). \tag{4.28}$$

Once again, we set  $p_5 = p_i = p_\phi = 0$ . Then from the normalization condition,

$$\dot{r}^2 = E^2 G(r) - F(r). \tag{4.29}$$

It is still true that  $R_{0'a0'a} = -\ddot{R}/R$  and  $R_{0'i0'i} = -\ddot{H}/H$  where  $a = 2, 3, 4$  and  $i = 6, \dots, 9$ . Near  $r = r_0$ , it is more convenient to write  $R = \tilde{f}_1^{1/4} \tilde{f}_5^{1/4} \sqrt{r_1 r_5}$  and  $H = \tilde{f}_1^{1/4} \tilde{f}_5^{-1/4} \sqrt{r_1/r_5}$ , where

$$\tilde{f}_1 = 1 + \frac{r^2}{r_1^2}, \quad \tilde{f}_5 = 1 + \frac{r^2}{r_5^2}. \tag{4.30}$$

Thus,

$$\begin{aligned}
R_{0'a0'a} &= -\frac{E^2}{R} (GR'' + G'R'/2) + \dots \\
&= -\frac{E^2}{16\tilde{f}_1^2 \tilde{f}_5^2} \left[ f_n (4\tilde{f}_1'' \tilde{f}_1 \tilde{f}_5^2 - 3\tilde{f}_1'^2 \tilde{f}_5^2 + \tilde{f}_1' \tilde{f}_5' \tilde{f}_1 \tilde{f}_5) + 2f_n' \tilde{f}_1 \tilde{f}_5^2 + (1 \leftrightarrow 5) \right] + \dots
\end{aligned} \tag{4.31}$$

Similarly,

$$R_{0'i0'i} = -\frac{E^2}{H}(GH'' + G'H'/2) + \dots \quad (4.32)$$

$$= -\frac{E^2}{16\tilde{f}_1^2\tilde{f}_5^2} \left[ 4f_n\tilde{f}_1''\tilde{f}_1\tilde{f}_5^2 + 2f_n'\tilde{f}_1'\tilde{f}_1\tilde{f}_5^2 - (1 \leftrightarrow 5) - f_n(2\tilde{f}_1'\tilde{f}_5'\tilde{f}_1\tilde{f}_5 + 3\tilde{f}_1'^2\tilde{f}_5^2 - 5\tilde{f}_5'^2\tilde{f}_1^2) \right] + \dots$$

where, as before, we have included only the terms proportional to  $E^2$ .

If we assume  $r_1, r_5 \gg r_0$ , the curvature at the horizon in the static frame is of order  $1/(r_1 r_5)$ , so we must take  $r_1 r_5 \gg 1$  to make this small. If  $r_0, r_n \ll r_1, r_5$ , then  $\tilde{f}_1, \tilde{f}_5 \approx 1$  at  $r = r_0$ , and the tidal forces are then not large. However, if  $r_0, r_n, r_1 \ll r_5$ , then the tidal forces are proportional to  $E^2/r_1^2$ , and so can be large if we take  $r_1 \ll 1$ . If we also take  $r_5 \gg 1/r_1$ , we can keep the curvature in the static frame small at the same time. This limit is physically reasonable, as we can suppose that  $V$  is sufficiently large to make  $Q_1$  and  $n$  large despite the fact that  $r_1, r_n \ll 1$ . Many of the recent calculations of near extremal black hole entropy and emission rates in string theory have assumed  $r_0, r_n \ll r_1, r_5$ . However, the case  $r_0, r_n, r_1 \ll r_5$  has been considered in e.g. [20,21].

## 5. Discussion

We have seen that objects falling into large black holes can experience large tidal forces outside the horizon. This seems to be a property of most of the recently discussed near extremal black hole and black p-brane solutions in string theory. The entropy of all of the solutions in the previous section has recently been understood in terms of a correspondence principle [22] which relates the black holes at strong coupling to the states of strings and D-branes at weak coupling. The transition occurs when the  $\alpha'$  corrections to the metric become significant. We have seen that this occurs when the size of the horizon is of order the string scale (so the static curvature is of order the string scale) and not when the tidal forces are large. Thus the agreement found in [22] between the counting of string states and black hole entropy is not affected by the results found here.

What are the physical effects of the large curvature outside the horizon? Consider a solid object falling toward the black hole. Tidal forces in the radial direction remain small, but those in the transverse directions become very large. When they become greater than the internal pressures and stresses can support, each particle essentially follows a geodesic. The object then shrinks to a fraction of its initial size before it crosses the horizon. In effect, the object is crushed by the gravitational field outside the black hole.



As mentioned in the introduction, the existence of large curvatures outside the horizon weakens the spirit of cosmic censorship. In light of this, it is natural to ask whether one could form these black holes from the collapse of charged matter with low density. Recall that neutral matter forms a Schwarzschild black hole when its average density is of order  $1/M^2$ , which is small for large  $M$ . Naked black holes typically have a horizon size which is much smaller than the Schwarzschild solution of the same mass. Nevertheless, it is still possible to form some of these black holes at low density. Consider the dilaton black holes discussed in section 3.1. In the near extremal limit,  $M = r_+/(1 + a^2)$ , so if this mass were confined to a radius of order the horizon size  $R = r_+ \epsilon^{a^2/(1+a^2)}$  it would have a density

$$\rho \sim \frac{r_+}{r_+^3 \epsilon^{3a^2/(1+a^2)}} = \frac{\epsilon^{(2-a^2)/(1+a^2)}}{(r_+ \epsilon)^2}. \quad (5.1)$$

The tidal forces on infalling observers are of order  $1/(r_+ \epsilon)^2$ , so for  $a^2 < 2$ , the parameters can be chosen so that the mass forms a black hole (with large tidal forces) when its average density is arbitrarily small. This includes the value  $a = 1$  which is important for string theory.

The existence of large tidal forces outside the horizon could play a role in resolving the black hole information puzzle for these black holes, since the large tidal forces may provide a mechanism for transferring information from the ingoing matter to the outgoing Hawking radiation. To examine this further, let us consider the calculation of Hawking radiation in one of these backgrounds. In the simplest case, one studies the propagation of a free scalar field in the black hole background, and computes the mixing of positive and negative frequency modes at past and future null infinity. Since the scalar wave equation can be evaluated in any frame, this equation does not see the large tidal forces. In other words, the effective radial potentials have a size set by the curvature in the static frame, which remains small.<sup>1</sup>

However, if we consider instead a test string falling into one of these black holes, the large tidal forces will cause it to become excited. So even if the string starts in a massless state at infinity, before it falls through the horizon, there is a high probability that it will be excited into a very massive state. This is similar to the effect on strings propagating through strong gravitational plane waves [17]. In fact, the physics is very similar, since the curvature is becoming large and almost null near the horizon.

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<sup>1</sup> This is different from the case studied in [23] where it was shown that these potentials can become large as the horizon shrinks to zero size for certain dilatonic black holes.

Proponents of unitarity have been looking for loopholes in Hawking's original arguments that information must be lost when black holes evaporate. The fact that the usual semi-classical calculations do not see the large tidal forces, while test strings do, may be an important clue to what is missing.

Even if these large tidal forces prove to be important for the information loss problem for these black holes, one still has to worry about black holes which are far from extremality, where the tidal forces remain small outside the horizon. It should be noted that even for Schwarzschild, one expects that string  $\alpha'$  and quantum corrections will destroy the boost invariance of the curvature. Thus generically, objects falling into a black hole feel different tidal forces depending on their energy. The tidal forces for Schwarzschild should remain small when the conserved energy  $E$  is of order one. However, one might speculate that some quantum analogs of this classical effect might resolve the information puzzle. Some potentially related mechanisms, which also depend on the difference between static and infalling observers, have been discussed in the context of string theory by Susskind and others [24].

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